RELATIVE EFFECT OF TEMPERATURE FACTOR ON TURBULENT GAS BOUNDARY LAYER AT FINITE REYNOLDS NUMBERS

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It is shown that when using the relations of turbulent boundary-layer theory for a fluid with vanishing viscosity the standard values of the friction coefficients and Stanton numbers are defined in terms of the Reynolds number

$$R_3^{**} = \frac{w_0 \delta^{**} \rho_0}{\mu_w}$$

In estimating the influence of the temperature factor on heat transfer the form in which the results are analyzed is of significant importance: in terms of the average flow parameters, as is done for pipe flow, or in terms of the parameters at the outer edge of the boundary layer.

Notations

c_f = friction coefficient; S, R, P, N = Stanton, Reynolds, Prandtl, Nusselt numbers respectively; ρ, γ, w, T, i = density, specific weight, velocity, temperature, enthalpy; ν, μ, λ = kinematic and dynamic viscosities and thermal conductivity; τ = tangential stress; $\delta, \delta^*, \delta^{**}, \delta_T^{**}$ = nominal, displacement, momentum loss, energy loss boundary layer thicknesses; y, ζ = transverse coordinates; ψ = temperature factor; D = pipe diameter.

Subscripts: w = quantities at the wall, 0 = conditions at the outer edge of the boundary layer, pipe centerline, standard conditions; $\langle\,\rangle$ = average parameters.

Turbulent boundary-layer theory yields a connection between the relative change of the friction and heat transfer coefficients and the temperature factor for $R \to \infty$ which is independent of the empirical "turbulence constants" [1-3].

In this case the question does not arise of determining the Reynolds number to which the standard quantities in the following functions are to be referred:

$$\Psi = \frac{c_f}{c_{f_0}}, \qquad \Psi_S = \frac{S}{S_0}$$

This is associated with the fact that

$$\left(\frac{\partial c_f}{\partial R}\right) \xrightarrow[R\to\infty]{} 0$$

and a large discrepancy in defining the Reynolds number does not reflect markedly on the values of c_{f_0} and S_0 for $R \to \infty$.

However, in the region of quite small R this situation no longer holds and the indeterminancy introduced by the possibility of referring the physical properties (ρ, μ) to different characteristic temperatures may be very large.

We shall examine this problem within the framework of asymptotic theory but with account for finiteness of the Reynolds numbers.

Novosibirsk. Translated from Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 11, No. 3, pp. 61-65, May-June, 1970. Original article submitted July 14, 1969.

As is known, as R $\to \infty$ there is degeneration of the viscous sublayer in the sense that its relative magnitude

$$\zeta_1 = (y_1/\delta) \to 0$$

Correspondingly, the temperature at the outer edge of the viscous sublayer

$$T_1 \to T_w$$

i.e., the larger the Reynolds number, the closer to isothermality is the viscous flow in the immediate vicinity of the wetted surface. Consequently, under these conditions the parameters of the viscous sublayer of the nonisothermal turbulent boundary layer are determined by the well known relations for standard conditions when referring the physical properties to the wall temperature.

As R $\to \infty$ we have

$$w_1 (\rho_w / \tau_w)^{1/2} \to y_1 (\tau_w / \rho_w)^{1/2} / \nu_w \to \eta_{10} \quad (1)$$

where, for example, in the two-layer model of the turbulent boundary layer $\eta_{10} = 11.6$. It is interesting to note that this result is in accord with the experimental data [5-7] even for not very large R.

For subsonic flow and sufficiently large R the function ψ has the form [2]

$$\Psi = \left[\frac{2(1 - \sqrt{\psi - (\psi - 1)\,\omega_1})}{z(\psi - 1)}\right]^2$$
$$\psi = \frac{T_w}{T_0}, \qquad z \to -2.5\sqrt{{}^1/{}_2 c_{f_0}} \ln \zeta_1 \quad (2)$$

Introducing here the values of ζ_1, from (1) we find that

$$z = z_1 \to z_0 - 2.5\sqrt{{}^1/{}_2 c_{f_0}} \ln \frac{\bar{\delta}^{**}}{\sqrt{\Psi}\sqrt{\psi}}$$
$$z_0 = 1 - \omega_{10}, \qquad R_1^{**} = \frac{w_0 \bar{\delta}^{**}}{\nu_w} \quad (3)$$

If we refer the physical properties to the temperature T_0 outside the boundary layer we have

$$R_2^{**} = \frac{w_0 \delta^{**}}{\nu_0}, \; z = z_2 \to z_0 - 2.5\sqrt{{}^1/{}_2 c_{f_0}} \ln \frac{\bar{\delta}^{**} \psi^{1/2 + m}}{\sqrt{\Psi}} \quad (4)$$

If we refer the dynamic viscosity to the wall temperature T_W and the density to the stream temperature T_0, as was done in [4], we have

$$R_3^{**} = \frac{w_0 \delta^{**} \rho_0}{\mu_w} \quad (5)$$

$$z = z_3 \to z_0 - 2.5\sqrt{{}^1/{}_2 c_{f_0}} \ln \frac{\bar{\delta}^{**}\sqrt{\psi}}{\sqrt{\Psi}} \quad (6)$$

In (4), m is the exponent of the temperature relation

$$\mu / \mu_0 = T / T_0)^m$$

Figure 1 shows the relation

$$\Psi^\circ = f(\psi), \; \Psi^\circ = \Psi / \Psi_\infty$$

Curves 1, 2, 3 were calculated using (2) and (6) for the values $R_3^{**} = 500, 10^4, 10^6$ respectively; curves 4, 5 were calculated using (2)-(4) for the values $R_1^{**} = R_2^{**} = 10^6$. We assumed similarity of the velocity and temperature fields, and the standard friction coefficient was found from the Karman equation

$$\sqrt{2 / c_{f_0}} = 2.5 \ln R^{**} + 3.8 \quad (7)$$

The quantity

$$\bar{\delta}^{**} = \left[\int_0^1 \frac{\rho}{\rho_0}\,\omega(1 - \omega)\,d\zeta\right]\left[\int_0^1 \omega(1 - \omega)\,d\zeta\right]^{-1}$$

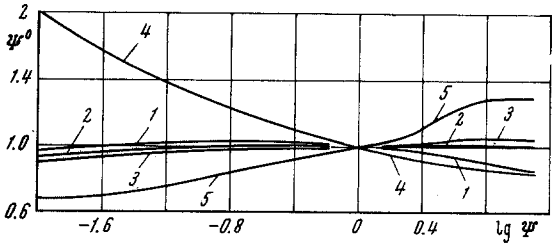

Fig. 1. Ratio Ψ/Ψ_{∞} as a function of ψ for various R^{**}: 1) $R_3^{**} = 500$; 2) 10^4; 3) 10^6; 4) $R_1^{**} = 10^6$; 5) $R_2^{**} = 10^7$.

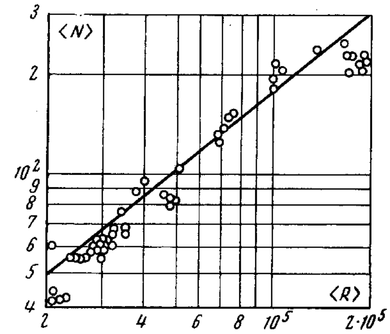

Fig. 2. Relation $\langle N\rangle = f(\langle R\rangle)$ for stabilized segment of pipe $T_W/\langle T\rangle = 0.2$–$0.7$: O) experiment; ——) from (10).

was calculated using the power law $\omega = \zeta^n$, where the quantity n was taken for the given R^{**} in converting (7) to the power friction law.

We see that if we select R^{**} in accordance with (5) the limiting friction law for subsonic flow

$$\Psi \to \left(\frac{2}{\sqrt{\psi}+1}\right)^2 (R^{**} \to \infty) \tag{8}$$

also describes satisfactorily the region of finite R. Here we emphasize that this definition of R^{**} should be considered not as a purely formal definition, but rather as an exact definition for large R.

We note that most of the experimental data from study of the effect of the temperature factor on heat transfer were obtained for conditions of gas flow in the stabilized segment of a pipe. In this case the gas viscosity and thermal conductivity were based on the average stream temperature; the temperature head was found as the difference between the average stream temperature and the wall temperature; the velocity which enters into R was taken as the discharge average velocity. Most of these experiments show weak influence of nonisothermality on the heat transfer or practically no such influence.

Data were obtained in [8] on the heat transfer under conditions of significant nonisothermality in the initial and stabilized segments of a cylindrical pipe. The experimental data were analyzed just as in the case of external flow, i.e., with respect to the parameters $\rho_0, w_0, \Delta T = T_0 - T_W$. It was shown that the relation

$$S = \frac{\Psi\, 0.0126}{(R^{**})^{0.25}\, P^{0.75}} \tag{9}$$

is valid for both the initial and stabilized segments, and the relation $\Psi/f(\psi)$ for these segments with account for (5) is close to the limit given by (8). Figure 2 shows these experimental data for the stabilized part of the pipe in the form of the average Nusselt number versus the average Reynolds number

$$\langle N\rangle = f(\langle R\rangle)$$

The numbers $\langle N\rangle$, $\langle R\rangle$ were found from the average values $\langle\rho\rangle$, $\langle w\rangle$, $\Delta T = \langle T\rangle - T_W$, μ and λ were based on $\langle T\rangle$. The values of $T_W/\langle T\rangle$ varied from 0.2 to 0.7. We see from Fig. 2 that in this form the effect of the temperature factor can be neglected.

This conclusion is also confirmed by the approximate theoretical calculation, in which the relation (9) for the stabilized segment was referred to the parameters $\langle\rho\rangle$, $\langle w\rangle$, $\Delta T = \langle T\rangle - T_W$, $\langle\mu\rangle$, $\langle\lambda\rangle$ in application to the experimental conditions of [8].

Assuming similarity of the velocities and enthalpies, we found the displacement and energy loss thicknesses

$$\frac{2\delta^*}{D} = \int_0^1 \left(1 - \frac{\rho w}{\rho_0 w_0}\right)(1-\zeta)\, d\zeta \qquad \left(\zeta = \frac{2y}{D}\right)$$

$$\frac{2\delta_T^{**}}{D} = \int_0^1 \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{i - i_w}{i_0 - i_w}\right)(1 - \zeta)\, d\zeta$$

where

$$\frac{\rho}{\rho_0} = \frac{1}{\psi - (\psi - 1)\zeta^{1/7}}, \qquad \omega = \frac{w}{w_0} = \frac{i - i_w}{i_0 - i_w} = \zeta^{1/7}$$

The value of $\langle \gamma w \rangle$ was found from the equation

$$\langle \gamma w \rangle = \gamma_0 w_0 \left(\frac{4\delta^*}{D}\right)$$

The average value $\langle T \rangle$ was found in terms of $\langle i \rangle$

$$\gamma_0 w_0 (i_0 - i_w)(D - 2\delta_T^{**})^2 = \langle \gamma w \rangle (\langle i \rangle - i_w) D^2$$

Knowing R^{**}, from (9) we can find the magnitude of the heat flux q and then the average value of the Nusselt number

$$\langle N \rangle = \frac{qD}{(\langle T \rangle - T_w)\langle \lambda \rangle}$$

The value of $\langle N_0 \rangle$ for isothermal conditions was found from the known equation

$$\langle N_0 \rangle = 0.018 \langle R \rangle^{0.8} \left(\langle R \rangle = \frac{\langle \gamma w \rangle D}{g \langle \mu \rangle}\right) \tag{10}$$

The calculations show that the values of $\langle N \rangle / \langle N_0 \rangle$ are smaller than Ψ in (9) by 10-15%.

In conclusion the authors wish to thank L. A. Bezuglova for assistance in carrying out the calculations.

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